HO-003-0492002

B. Sc./M. Sc. (Sem. II) (CBCS) Examination

April - 2023

Applied Mathedmatics : Paper-VI

(Applied Physics(

(New Course)

Faculty Code : 003
Subject Code : 0492002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instruction : (1) All questions are compulsory.
(2) Numbers in the right margin indicate marks.

1 (a) Attempt short questions : (**One** mark each)   **4**

     (1) Solve : $xdy–ydx=0$

     (2) Find the I.F. for $y'+\dfrac{1}{x}y=1$.

     (3) Is the differential equation $(x^2 - ay)dx + (y^2 - ax)dy = 0$ exact ?

     (4) Solve : $y=px+log\ p$.

(b) Write answers of any **One** :   **2**

     (1) Solve : $(xy^2 + x)dx + (x^2y + y)dy = 0$

     (2) From differential equation for the curve $y^2 = ax + b$.

(c) Answer any **one** of the following :   **3**

     (1) Solve : $(x^2 + y^2)dx + 2x^2dy = 0$

     (2) Solve : $y'-\dfrac{2y}{x+1} = x+1$.

(d) Write answers of any **One**.   **5**

     (1) Solve : $y - 2px = \tan^{-1}(xp^2)$.

     (2) Solve : $x = py + p^2y^2$.

2 (a) Attempt short questions : (**One mark each**)   **4**

      (1) If $z = x^3 + y^3 - 3axy$ then find z_x.

      (2) If $x = u(1-v), y = v(1-u)$ then find $J\left(\dfrac{x,y}{u,v}\right)$.

      (3) If $z = x^{-1} + y^{-1}$ then prove that $xz_x + yz_y = -z$.

      (4) Find the critical points for the surface $z = x^2 - x + y^2$

   (b) Write answers of any **One** :   **2**

      (1) If $z = x^y$ then find z_{xy}.

      (2) If $z = \dfrac{x}{y} + \dfrac{y}{x}$ then prove that $xz_x + yz_y = 0$.

   (c) Write answers of any **One** :   **3**

      (1) Expand $e^x \sin x$ in infinite terms.

      (2) Expand sin^2x in infinite terms.

   (d) Write answers of any **One** :   **5**

      (1) Find the shortest distance from origin to hyperbola

           $xy = c^2$

      (2) Find extrema of $z = x^2 + 2y^2 - x$.

3 (a) Attempt short questions : (**One mark each**)   **4**

      (1) Form P. D. E. for $z = f(x^2 + y^2)$.

      (2) Form $z_{xy} = 2y + 1$.

      (3) Solve : $p + q = 1$.

      (4) Solve : $z = px + qy + pq$.

   (b) Write answers of any **One** :   **2**

      (1) Form P. D. E. for $z = (x+a)(y+b)$

      (2) Solve : $xp + yq = z$.

   (c) Write answers of any **One** :   **3**

      (1) Solve : $p + q = x + y$.

      (2) Solve : $py^3 + qx^2 = 0$.

   (d) Write answers of any **One** :   **5**

      (1) Solve : $z_{xx} - z_{xy} = \cos x$.

      (2) Solve : $z_{xx} - 2z_x + z_y = 0$

4   (a)   Attempt short questions : (**One mark each**)   **4**

        (1)   Verify Roll's theorem for $f(x) = x^2 - x, x \in [0,1]$.

        (2)   Using infinite expansion of e^x, deduce the series for e.

        (3)   Show that $f(x) = e^x, x \in R$ is an increasing function.

        (4)   Evaluate : $\int_0^1 \int_{-1}^1 x \, dx \, dy$

   (b)   Write answers of any **One** :   **2**

        (1)   Interpret Lagrang's theorem geometrically.

        (2)   Find c, using Lagrange's Mean Value theorem for
           $f(x) = \log x, x \in [1,e]$.

   (c)   Write answers of any **One** :   **3**

        (1)   Show that $x > \log(x+1)$ for $x \in R^+$

        (2)   Expand $f(x)=tanx$ (two terms)

   (d)   Write answers of any **One** :   **5**

        (1)   Evaluate : $\iint_R (x^2 + y^2) dx \, dy$ where R is $x^2 + y^2 \le 1$.

        (2)   Evaluate : $\iiint_R dx \, dy \, dz$ where R is the unit sphere.

5   (a)   Attempt short questions : (**One mark each**)   **4**

        (1)   Form differential equation for $y^2 = ax$.

        (2)   Solve : $xdy + ydx = 0$.

        (3)   If $x=uv, y=u+v$ find $J\left(\dfrac{x,y}{u,v}\right)$.

        (4)   Solve : $z=px+qy+p+q$.

   (b)   Write answers of any **One** :   **2**

        (1)   Solve : $p=sin\ (y-xp)$.

        (2)   If $x^{-1} + y^{-1} - z^{-1} = 0$ then find z_x.

   (c)   Write Answer of any **one** :   **3**

        (1)   Interpret Roll's theorem geometrically.

        (2)   Expand $\cos^2 x$ in infinite series.

   (d)   Write Answer of any **one** :   **5**

        (1)   Evaluate : $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx \, dy$.

        (2)   Evaluate : $\iint_R dx \, dy$ where R is the region enclosed by
           $y = x^2$ and $y = x$.